Analysis of $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge theory

William A. Ponce
Instituto de Física, Universidad de Antioquia,
A.A. 1226, Medellín, Colombia.

Juán B. Flórez
Depto. de Física, Universidad de Nariño
Pasto, Colombia.

Luis A. Sánchez
Escuela de Física, Universidad Nacional de Colombia
A.A. 3840, Medellín, Colombia.

Abstract

Six different models, straightforward extensions of the standard model to $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, which do not contain particles with exotic electric charges are presented. Two of the models are one family and four are three family models. In two of the three family models one of the families transforms different from the others, and in the other two all the three families are different.

1 Introduction

The remarkable experimental success of the standard model (SM) local gauge group $G_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with the flavor sector $SU(2)_L \otimes U(1)_Y$ hidden[1] and $SU(3)_c$ confined[2], lies in its accurate predictions at energies below a few hundreds GeV. However, the SM is not the only model for which this is true and many physicists believe that it does not represent the final theory, but serves merely as an effective theory originating from a more fundamental one. So, extensions of the SM are always welcome.

One can extend the SM either by adding new fermion fields (adding a right-handed neutrino field constitute its simplest extension), by augmenting the scalar sector to more than one higgs representation, or by enlarging the local gauge group. In this last direction, $SU(3)_L \otimes U(1)_X$ as a flavor group has been studied previously by many authors[3] who have explored many possible fermion and higgs-boson representation assignments, either as identical replicas of one family structures as in the SM [4] or as a multi-family structure[5, 6] which points to a natural explanation of the total number of families in nature.

With regard to the different models in Ref.[4], most of them are plagued with physical inconsistencies such as gauge anomalies, right-handed currents at low energies, unwanted flavor changing neutral currents, violation of universality, etc.. The model in Refs.[5] for three families of quarks and leptons is consistent with the low energy phenomenology and it is anomaly free thanks to the introduction of quarks with exotic electric charges $-4/3$ and $5/3$. On the other hand, the model in Refs.[6], also for three families, is consistent with low energy phenomenology and does not include particles with exotic electric charges.

In this paper we present an analysis of the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. We find six models which are anomaly free, do not include fermions (quarks and leptons) with exotic electric charges and are consistent with the low energy phenomenology. Two of the models are one family models and are natural extensions of the SM (one of them is an E_6 subgroup), while the other four are models for three families of quarks and leptons; two of them, up to our knowledge, new in the literature. The models under consideration get their

symmetries broken via the most economical set of higgs fields. We analyze also the limit in which the neutral currents reproduce the SM phenomenology.

Our paper is organized in the following way: In section two we introduce the characteristics of the gauge group and present the six different models mentioned above; in section three we describe the scalar sector needed to break the symmetry; in section four we analyze the gauge boson sector paying special attention to the two neutral currents and their mixing, in section five we analyze the fermion masses for one particular model and in the last section we give our conclusions.

2 The model

In what follows we assume that the electroweak gauge group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$. We also assume that the left handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two fundamental representations of $SU(3)_L$ (the 3 and 3^*). Two classes of models will be discussed: one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the families. As in the SM, $SU(3)_c$ is vectorlike.

All the models analyzed have the same gauge boson sector, but they differ in their fermion content and may differ in the scalar sector too.

2.1 One family models

The most general expression for the electric charge generator in $SU(3)_L \otimes U(1)_X$ is a linear combination of the three diagonal generators of the gauge group

$$Q = aT_{3L} + \frac{2}{\sqrt{3}} bT_{8L} + XI_3, \tag{1}$$

where $T_{iL} = \lambda_{iL}/2$; λ_{iL} being the Gell-Mann matrices for $SU(3)_L$ normalized as $\mathbf{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}$, $I_3 = Dg(1,1,1)$ is the diagonal 3×3 unit matrix, and a and b are arbitrary parameters to be calculated ahead. Notice that we have absorbed an eventual coefficient for X in its definition.

Now, having in mind the canonical iso-doublets for $SU(2)_L$ in one family, we start by defining two $SU(3)_L$ triplets

$$\chi_L = \begin{pmatrix} u \\ d \\ q \end{pmatrix}_L , \qquad \psi_L = \begin{pmatrix} e^- \\ \nu_e \\ l \end{pmatrix}_L ,$$

where q_L and l_L are $SU(2)_L$ singlet exotic quark and lepton fields respectively of electric charges to be fixed ahead. This structure implies that $a = 1$ in Eq.(1) and one gets a one-parameter set of models. Now if the $\{SU(3)_L,\ U(1)_X\}$ quantum numbers for χ_L and ψ_L are $\{3, X_\chi\}$ and $\{3^*, X_\psi\}$ respectively, then by using Eq.(1) we have the relationship:

$$X_\chi + X_\psi = Q_q + Q_l = -1/3, \tag{2}$$

where Q_q and Q_l are the electric charge values of the $SU(2)_L$ singlets q and l respectively, in units of the absolute value of the electron electric charge.

Now in order to cancel the $[SU(3)_L]^3$ anomaly, two more $SU(3)_L$ lepton anti-triplets with quantum numbers $\{3^*, X_i\}$, $i = 1, 2$, must be introduced (together with their corresponding right-handed charged components). Each one of those multiplets must include one $SU(2)_L$ doublet and one singlet of new leptons. The quarks fields u^c_L, d^c_L and q^c_L color anti-triplets and $SU(3)_L$ singlets, with $U(1)_X$ quantum numbers X_u, X_d and X_q respectively, must also be introduced in order to cancel the $[SU(3)_c]^3$ anomaly. Then the hypercharges X_α with $\alpha = \chi, \psi, 1, 2, u, d, q, ...$ are fixed using Eqs. (1), (2) and the anomaly constraint equations coming from the vertices $[SU(3)_c]^2U(1)_X$, $[SU(3)_L]^2U(1)_X$, $[grav]^2U(1)_X$ and $[U(1)_X]^3$, which are:

$$\begin{aligned}
[SU(3)_c]^2U(1)_X &: \; 3X_\chi + X_u + X_d + X_q = 0 \\
[SU(3)_L]^2U(1)_X &: \; 3X_\chi + X_\psi + X_1 + X_2 = 0 \\
[grav]^2U(1)_X &: \; 9X_\chi + 3X_u + 3X_d + 3X_q + 3X_\psi + 3X_1 + 3X_2 + \sum_{singl} X_{ls} = 0 \\
[U(1)_X]^3 &: \; 9X_\chi^3 + 3X_u^3 + 3X_d^3 + 3X_q^3 + 3X_\psi^3 + 3X_1^3 + 3X_2^3 + \sum_{singl} X_{ls}^3 = 0,
\end{aligned}$$

where X_{ls} are the hypercharges of the right-handed charged lepton singlets needed in order to have a consistent field theory.

What we have so far is an infinite number of possible models each one characterized by the parameter b in Eq.(1); the value of b is the key factor in determining the electric charge of the extra particles in the several models to be presented. We are going to drastically limit this number of possible models by imposing the constraint of excluding models with particles with exotic electric charges; that is, we are going to allow only models with quarks of electric charges $\pm 2/3$ and $\pm 1/3$ and leptons of electric charges ± 1 and 0. We will see that this requirement render us with only two different sets of models ($b = \pm 1/2$) with equivalent gauge sectors.

2.1.1 Model A

Let us start with a model with an extra down type quark $q = D$ of electric charge $Q_q = Q_D = -1/3$ ($b = 1/2$) which in turn implies $Q_l = 0$, that is, l_L is a new neutral lepton N^0_{1L}. Eq.(1) then implies $X_q = X_d = 1/3$, $X_u = -2/3$, which combined with the anomaly constraint equations and Eq.(2) gives $X_\chi = 0$, $X_\psi = -1/3$, $\sum_{singl} X_{ls} = 0$ and $X_1 + X_2 = 1/3$. By demanding for leptons of electric charges ± 1 and 0 only, we have for the simplest solution that $X_1 = -1/3$, $X_2 = 2/3$ and $X_{ls} = 0$, with this last constraint meaning that we do not need right-handed charged leptons in our simplest anomaly-free model.

Putting all this together we end up with the following multiplet structure for this model:

$\chi_L = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_L$	u^c_L	d^c_L	D^c_L
$(3, 3, 0)$	$(3^*, 1, -\frac{2}{3})$	$(3^*, 1, \frac{1}{3})$	$(3^*, 1, \frac{1}{3})$

$\psi_L = \begin{pmatrix} e^- \\ \nu_e \\ N_1^0 \end{pmatrix}_L$	$\psi_{1L} = \begin{pmatrix} E^- \\ N_2^0 \\ N_3^0 \end{pmatrix}_L$	$\psi_{2L} = \begin{pmatrix} N_4^0 \\ E^+ \\ e^+ \end{pmatrix}_L$
$(1, 3^*, -\frac{1}{3})$	$(1, 3^*, -\frac{1}{3})$	$(1, 3^*, \frac{2}{3})$

,

where the numbers inside the parenthesis refer to $(SU(3)_c, SU(3)_L, U(1)_X)$ quantum numbers. This anomaly-free structure is the simplest one we can construct for a single family in $SU(3)_L \otimes U(1)_X$. As a matter of fact, the 27 states above are just the 27 states in the fundamental representation of the electroweak-strong unification group E_6[7], so this gauge and fermion structure is such that $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \subset E_6$. A phenomenological analysis of this model has been published already in Ref.[8].

2.1.2 Model B

For this model we start with an extra up type quark $q = U$ of electric charge $Q_q = Q_U = 2/3(b = -1/2)$ which in turn implies $Q_l = -1$, that is, l_L is now an exotic electron E^-. Following the same steps as for model **A** we end up with the following multiplet structure:

$\chi_L = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L$	d_L^c	u_L^c	U_L^c
$(3, 3, \frac{1}{3})$	$(3^*, 1, \frac{1}{3})$	$(3^*, 1, -\frac{2}{3})$	$(3^*, 1, -\frac{2}{3})$

$\psi_L = \begin{pmatrix} e^- \\ \nu_e \\ E_1^- \end{pmatrix}_L$	$\psi_{1L} = \begin{pmatrix} N_1^0 \\ E_2^+ \\ \nu_e^c \end{pmatrix}_L$	$\psi_{2L} = \begin{pmatrix} E_2^- \\ N_2^0 \\ E_3^- \end{pmatrix}_L$	e_L^+	E_{1L}^+	E_{3L}^+
$(1, 3^*, -\frac{2}{3})$	$(1, 3^*, \frac{1}{3})$	$(1, 3^*, -\frac{2}{3})$	$(1, 1, 1)$	$(1, 1, 1)$	$(1, 1, 1)$

A simple check shows that this multiplet structure is also free of anomalies. A phenomenological analysis of this model has been started already in Ref.[9],

where it is shown that model **B** as presented here is a subgroup of $SU(6)\otimes U(1)$, an electroweak-strong unification group which has not been considered in the literature so far.

The gauge boson content of models **A** and **B** are equivalent, and they become the same just by replacing $3 \leftrightarrow 3^*$ in the irreducible representations of the fermion fields ($b \rightarrow -b$ when the complex conjugate of the covariant derivative is taken).

2.1.3 Other one-family Models

Following the same steps as for the two previous cases, we attempt to construct models where q_L has electric charges $-2/3$ or $1/3$. Eq.(2) then implies that $Q_l = 1/3$ and $-2/3$ respectively which correspond to leptons with exotic electric charges. Not only fractionally charged free particles has not been detected at low energies, but the phenomenology of those models could become tremendously confusing with leptons with electric charges equal to the antiquarks.

In a similar way by asking for a model with $Q_l = 1$ we will have, according to Eq.(2), that $Q_q = -4/3$, a model with a quark with an exotic electric charge which we have excluded from the models discussed here (a model with quarks with exotic electric charges is presented in Ref.[5] for example).

2.2 Family models

For these models each individual family possesses non-vanishing anomalies and the anomaly cancellation takes place between families and, for some models, only with a matching of the number of families with the number of quark colors, does the overall anomaly vanish[5, 6, 10]. It is also a feature of this type of models that the third family is treated different to the other two, or either that all the three families are treated independently.

An algebraic manipulation of Eqs.(1) and (2) and the anomaly constraint equations, allows us to combine the fermion multiplets of the two models **A** and **B** to produce the following models (with the replacement $3 \leftrightarrow 3^*$ in model **B** in order to assure a unique covariant derivative):

2.2.1 Model C

All the left-handed lepton generations belong to the representation $(1, 3, -2/3)$ of $(SU(3)_c, SU(3)_L, U(1)_X)$, that is:

$\psi_L^\alpha = \begin{pmatrix} \nu_\alpha \\ \alpha^- \\ E_\alpha^- \end{pmatrix}_L$	α_L^+	$E_{\alpha L}^+$
$(1, 3, -2/3)$	$(1, 1, 1)$	$(1, 1, 1)$

for $\alpha = e, \mu, \tau$; while quarks transform as follows:

$\chi_L^a = \begin{pmatrix} d^a \\ u^a \\ U^a \end{pmatrix}_L$	u_L^{ac}	d_L^{ac}	U_L^{ac}
$(3, 3^*, 1/3)$	$(3^*, 1, -\frac{2}{3})$	$(3^*, 1, \frac{1}{3})$	$(3^*, 1, -\frac{2}{3})$

for $a = 1, 2$ two of the families. For the other family we have:

$\chi_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ D \end{pmatrix}_L$	u_{3L}^c	d_{3L}^c	D_L^c
$(3, 3, 0)$	$(3^*, 1, -\frac{2}{3})$	$(3^*, 1, \frac{1}{3})$	$(3^*, 1, \frac{1}{3})$

.

The arithmetic shows that all the anomalies vanish for this fermion content. As far as we know the study of this model is relatively new in the literature; it was introduced for the first time in Ref.[10].

At first glance this structure does not allow for neutrino masses; even though, a variant of this model, with the capability to explain the main features of the atmospheric and solar neutrino experimental results, has been presented in Ref.[11].

2.2.2 Model D

In a similar way we get the following multiplet structure:

$\chi_L^a = \begin{pmatrix} u_a \\ d_a \\ D_a \end{pmatrix}_L$	u_{aL}^c	d_{aL}^c	D_{aL}^c
$(3,3,0)$	$(3^*,1,-\frac{2}{3})$	$(3^*,1,\frac{1}{3})$	$(3^*,1,\frac{1}{3})$

for $a = 1, 2$, the quarks in two of the three families. For the quarks in the other family we have:

$\chi_L^3 = \begin{pmatrix} d_3 \\ u_3 \\ U \end{pmatrix}_L$	u_{3L}^c	d_{3L}^c	U_L^c
$(3,3^*,\frac{1}{3})$	$(3^*,1,-\frac{2}{3})$	$(3^*,1,\frac{1}{3})$	$(3^*,1,-\frac{2}{3})$

.

The three lepton generations transform now as anti-triplets of $SU(3)_L$ as follows:

$\psi_L^\alpha = \begin{pmatrix} \alpha^- \\ \nu_\alpha \\ N_\alpha^0 \end{pmatrix}_L$	α_L^+
$(1,3^*,-\frac{1}{3})$	$(1,1,1)$

,

for $\alpha = e, \mu, \tau$ the three families. This model has been largely studied in the literature (see Refs.[6]). Again, this model has been used recently in connection with neutrino oscillations[12].

2.2.3 Other models

Contrary to the one family models, we can now play the game of canceling the anomalies in several different ways.

We start by defining the following closed set of fermions (closed in the sense that they include the antiparticles of the charged particles):
$S_1 = [(\nu_\alpha, \alpha^-, E_\alpha^-); \alpha^+; E_\alpha^+]$ with quantum numbers $(1,3,-2/3); (1,1,1)$ and $(1,1,1)$ respectively.

$S_2 = [(\alpha^-, \nu_\alpha, N^0_\alpha); \alpha^+]$ with quantum numbers $(1, 3^*, -1/3)$ and $(1, 1, 1)$ respectively.
$S_3 = [(d, u, U); u^c; d^c; U^c]$ with quantum numbers $(3, 3^*, 1/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, -2/3)$ respectively.
$S_4 = [(u, d, D); d^c; u^c; D^c]$ with quantum numbers $(3, 3, 0)$; $(3^*, 1, 1/3)$; $(3^*, 1, -2/3)$ and $(3^*, 1, 1/3)$ respectively.
$S_5 = [(e^-, \nu_e, N^0_1); (E^-, N^0_2, N^0_3); (N^0_4, E^+, e^+)]$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, -1/3)$ and $(1, 3^*, 2/3)$ respectively.
$S_6 = [(\nu_e, e^-, E^-); (E^+_2, N^0_1, N^0_2); (N^0_3, E^-_2, E^-_3); e^+, E^+_1; E^+_3]$ with quantum numbers $(1, 3, -2/3)$; $(1, 3, 1/3)$; $(1, 3, -2/3)$; $(1, 1, 1)$; $(1, 1, 1)$ and $(1, 1, 1)$ respectively.

Now we calculate the four anomalies for each set of particles. The results are presented in Table I.

TABLE I. Anomalies for S_i.

Anomalies	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_c]^2 U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	$-2/3$	$-1/3$	1	0	0	-1
$[grav]^2 U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	$10/9$	$8/9$	$-12/9$	$-6/9$	$6/9$	$12/9$

Notice from Table I that model **A** is just $(S_4 + S_5)$ and model **B** is $(S_3 + S_6)$. Model **C** is represented by $(3S_1 + 2S_3 + S_4)$ and model **D** by $(3S_2 + S_3 + 2S_4)$, but what is most remarkable is that we can now construct new anomaly-free models for two, three, four and more families. For example two new three family models are:

Model E: $S_1 + S_2 + S_3 + S_4$ plus Model **A** $= (S_1 + S_2 + S_3 + 2S_4 + S_5)$
Model F $S_1 + S_2 + S_3 + S_4$ plus Model **B**$=(S_1 + S_2 + 2S_3 + S_4 + S_6)$.
A model for four families will be given for example by: $2(S_1 + S_2 + S_3 + S_4)$, etc..

The main feature of models **E** and **F** above is that, contrary to models **C** and **D**, each one of the three families is treated in a different way. As far as we know, these two models have not been studied in the literature so far.

3 The scalar sector

Even though the representation content for the fermions may vary significantly from model to model, all $SU(3)_L \otimes U(1)_X$ models presented so far have the same gauge boson sector as it will be discussed in the following section. Now, to achieve an spontaneous breaking of the symmetry in the most economic way, using the chain

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \longrightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_c \otimes U(1)_Q,$$

we need two complex higgs scalars $\phi_i(1, 3^*, -1/3) = (\phi_i^-, \phi_i^0, \phi_i^{\prime 0}),\ i = 1, 2$, with Vacuum Expectation Values (VEV) $\langle\phi_1\rangle = (0, 0, V)^T$ and $\langle\phi_2\rangle = (0, v/\sqrt{2}, 0)^T$, with the hierarchy $V >> v \sim 250$ GeV the electroweak mass scale. Now, to break the symmetry and at the same time give masses to all the fermion fields is a model dependent analysis. So, let us outline in the following three sections the analysis for model **A** for example, for which a third higgs field $\phi_3(1, 3^*, 2/3) = (\phi_3^0, \phi_3^+, \phi_3^{\prime +})$ with VEV $\langle\phi_3\rangle = (v'/\sqrt{2}, 0, 0)^T$ is also needed, where $v' \simeq v$[8].

The analysis for model **B** is done in Ref.[9] and for model **D** in Ref.[6].

4 The gauge boson sector

There are a total of 17 gauge bosons in the gauge group under consideration; they are: one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 8 associated with $SU(3)_L$ and that we write for convenience in the following way:

$$\frac{1}{2}\lambda_\alpha A_\alpha^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} D_1^\mu & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D_2^\mu & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D_3^\mu \end{pmatrix},$$

where $D_1^\mu = A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6},\ D_2^\mu = -A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6}$, and $D_3^\mu = -2A_8^\mu/\sqrt{6}$. $\lambda_i,\ i = 1, 2, ..., 8$ are the eight Gell-Mann matrices normalized as mentioned in

Section **2.1** This allows us to write now the charge operator as

$$Q = \frac{\lambda_3}{2} + \frac{\lambda_8}{2\sqrt{3}} + XI_3,$$

where I_3 is the 3×3 unit matrix.

In model **A**, after breaking the symmetry with $\langle\phi_i\rangle$, $i = 1, 2, 3$, and using for the covariant derivative for triplets $D^\mu = \partial^\mu - i\frac{g}{2}\lambda_\alpha A^\mu_\alpha - ig'XB^\mu$, we get the following mass terms for the charged gauge bosons in the electroweak sector: $M^2_{W^\pm} = \frac{g^2}{4}(v^2 + v'^2)$, $M^2_{K^\pm} = \frac{g^2}{4}(2V^2 + v'^2)$, $M^2_{K^0(\bar{K}^0)} = \frac{g^2}{4}(2V^2 + v^2)$. For the neutral gauge bosons we get a mass term of the form:

$$M = V^2(\frac{g'B^\mu}{3} - \frac{gA^\mu_8}{\sqrt{3}})^2 + \frac{v^2}{8}(\frac{2g'B^\mu}{3} - gA^\mu_3 + \frac{gA^\mu_8}{\sqrt{3}})^2 + \frac{v'^2}{8}(gA^\mu_3 - \frac{4g'B^\mu}{3} + \frac{gA^\mu_8}{\sqrt{3}})^2$$

By diagonalizing M we get the physical neutral gauge bosons which are defined through the mixing angle θ and Z_μ, Z'_μ by:

$$\begin{aligned} Z^\mu_1 &= Z_\mu \cos\theta + Z'_\mu \sin\theta \\ Z^\mu_2 &= -Z_\mu \sin\theta + Z'_\mu \cos\theta \\ -\tan(2\theta) &= \frac{\sqrt{12}C_W(1 - T^2_W/3)^{1/2}[v'^2(1 + T^2_W) - v^2(1 - T^2_W)]}{3(1 - T^2_W/3)(v^2 + v'^2) - C^2_W[8V^2 + v^2(1 - T^2_W)^2 + v'^2(1 + T^2_W)^2]}, \end{aligned} \tag{3}$$

where the photon field A^μ and the fields Z_μ and Z'_μ are given by

$$\begin{aligned} A^\mu &= S_W A^\mu_3 + C_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1 - T^2_W/3)^{1/2}B^\mu\right], \\ Z^\mu &= C_W A^\mu_3 - S_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1 - T^2_W/3)^{1/2}B^\mu\right], \\ Z'^\mu &= -(1 - T^2_W/3)^{1/2}A^\mu_8 + \frac{T_W}{\sqrt{3}}B^\mu. \end{aligned} \tag{4}$$

S_W and C_W are the sine and cosine of the electroweak mixing angle respectively ($T_W = S_W/C_W$) defined by $S_W = \sqrt{3}g'/\sqrt{3g^2 + 4g'^2}$. Also we can identify the Y hypercharge associated with the SM gauge boson as:

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1 - T^2_W/3)^{1/2}B^\mu\right].$$

In the limit $\theta \longrightarrow 0$, $M_Z = M_{W^\pm}/C_W$, and $Z_1^\mu = Z^\mu$ is the gauge boson of the SM. This limit is obtained either by demanding $V \longrightarrow \infty$ or $v'^2 = v^2(C_W^2 - S_W^2)$. In general θ may be different from zero although it takes a very small value, determined from phenomenology for each particular model.

The former definitions for A^μ, Z^μ, Z'^μ, Y^μ and S_W are the same for all the six models in Section **2**. The value for $\tan(2\theta)$ and the expressions for the masses of the gauge bosons are model dependent.

4.1 Currents

The currents for fermions are different for each model and also they are different from those of the SM. As an example let us present the analysis for model **A**[8]; a similar analysis for model **B** is presented in Ref.[9] and for model **D** in Ref.[6].

4.1.1 Charged currents

The interactions among the charged vector fields with leptons for model **A** are

$$\begin{aligned} H^{CC} &= \frac{g}{\sqrt{2}}[W_\mu^+(\bar{u}_L\gamma^\mu d_L - \bar{\nu}_{eL}\gamma^\mu e_L^- - \bar{N}_{2L}^0\gamma^\mu E_L^- - \bar{E}_L^+\gamma^\mu N_{4L}^0) \\ &\quad + K_\mu^+(\bar{u}_L\gamma^\mu D_L - \bar{N}_{1L}^0\gamma^\mu e_L^- - \bar{N}_{3L}^0\gamma^\mu E_L^- - \bar{e}_L^+\gamma^\mu N_{4L}^0) \\ &\quad + K_\mu^0(\bar{d}_L\gamma^\mu D_L - \bar{N}_{1L}^0\gamma^\mu \nu_{eL} - \bar{N}_{3L}^0\gamma^\mu N_{2L}^0 - \bar{e}_L^+\gamma^\mu E_L^+)] + H.c., \qquad (5) \end{aligned}$$

which implies that the interactions with $K^\pm$ and $K^0(\bar{K}^0)$ bosons violate the lepton number and the weak isospin. Notice also that the first two terms in the previous expression constitute the charged weak current of the SM as far as we identify $W^\pm$ as the $SU(2)_L$ charged left-handed weak bosons.

4.1.2 Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$ and $J_\mu(Z')$, associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + \frac{g}{C_W}Z^\mu J_\mu(Z) + \frac{g'}{\sqrt{3}}Z'^\mu J_\mu(Z')$ are:

$$J_\mu(EM) = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{D}\gamma_\mu D - \bar{e}^-\gamma_\mu e^- - \bar{E}^-\gamma_\mu E^- = \sum_f q_f \bar{f}\gamma_\mu f$$

$$\begin{aligned} J_\mu(Z) &= J_{\mu,L}(Z) - S_W^2 J_\mu(EM) \\ J_\mu(Z') &= T_W J_\mu(EM) - J_{\mu,L}(Z'), \end{aligned} \tag{6}$$

where $e = gS_W = g'C_W\sqrt{1-T_W^2/3} > 0$ is the electric charge, q_f is the electric charge of the fermion f in units of e, $J_\mu(EM)$ is the electromagnetic current (vectorlike as it should be), and the left-handed currents are

$$\begin{aligned} J_{\mu,L}(Z) &= \frac{1}{2}(\bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L + \bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \bar{e}_L^-\gamma_\mu e_L^- + \bar{N}_2^0\gamma_\mu N_2^0 - \bar{E}^-\gamma_\mu E^-) \\ &= \sum_f T_{3f}\bar{f}_L\gamma_\mu f_L \\ J_{\mu,L}(Z') &= S_{2W}^{-1}(\bar{u}_L\gamma_\mu u_L - \bar{e}_L^-\gamma_\mu e_L^- - \bar{E}_L^-\gamma_\mu E_L^- - \bar{N}_{4L}^0\gamma_\mu N_{4L}^0) \\ &\quad T_{2W}^{-1}(\bar{d}_L\gamma_\mu d_L - \bar{E}_L^+\gamma_\mu E_L^+ - \bar{\nu}_{eL}\gamma_\mu\nu_{eL} - \bar{N}_{2L}^0\gamma_\mu N_{2L}^0) \\ &\quad -T_W^{-1}(\bar{D}_L\gamma_\mu D_L - \bar{e}_L^+\gamma_\mu e_L^+ - \bar{N}_{1L}^0\gamma_\mu N_{1L}^0 - \bar{N}_{3L}^0\gamma_\mu N_{3L}^0) \\ &= \sum_f T_{9f}\bar{f}_L\gamma_\mu f_L, \end{aligned} \tag{7}$$

where $S_{2W} = 2S_WC_W$, $T_{2W} = S_{2W}/C_{2W}$, $C_{2W} = C_W^2 - S_W^2$, $\bar{N}_2^0\gamma_\mu N_2^0 = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 + \bar{N}_{2R}^0\gamma_\mu N_{2R}^0 = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 - \bar{N}_{2L}^{0c}\gamma_\mu N_{2L}^{0c} = \bar{N}_{2L}^0\gamma_\mu N_{2L}^0 - \bar{N}_{4L}^0\gamma_\mu N_{4L}^0$, and similarly $\bar{E}\gamma_\mu E = \bar{E}_L^-\gamma_\mu E_L^- - \bar{E}_L^+\gamma_\mu E_L^+$. In this way $T_{3f} = Dg.(1/2, -1/2, 0)$ is the third component of the weak isospin acting on the representation 3 of $SU(3)_L$ (the negative when acting on 3^*), and $T_{9f} = Dg.(S_{2W}^{-1}, T_{2W}^{-1}, -T_W^{-1})$ is a convenient 3×3 diagonal matrix acting on the representation 3 of $SU(3)_L$ (the negative when acting on 3^*). Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM, which allows us to identify Z_μ as the neutral gauge boson of the SM.

The couplings of the physical states Z_1^μ and Z_2^μ are then given by:

$$\begin{aligned} H^{NC} &= \frac{g}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5) + a_{iR}(f)(1+\gamma_5)]f\} \\ &= \frac{g}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu[g(f)_{iV} - g(f)_{iA}\gamma_5]f\}, \end{aligned} \tag{8}$$

where

$$
\begin{aligned}
a_{1L}(f) &= \cos\theta(T_{3f} - q_f S_W^2) - \frac{g'\sin\theta C_W}{g\sqrt{3}}(T_{9f} - q_f T_W) \\
a_{1R}(f) &= -q_f S_W(\cos\theta S_W - \frac{g'\sin\theta}{g\sqrt{3}}) \\
a_{2L}(f) &= -\sin\theta(T_{3f} - q_f S_W^2) - \frac{g'\cos\theta C_W}{g\sqrt{3}}(T_{9f} - q_f T_W) \\
a_{2R}(f) &= q_f S_W(\sin\theta S_W + \frac{g'\cos\theta}{g\sqrt{3}}),
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
g(f)_{1V} &= \cos\theta(T_{3f} - 2S_W^2 q_f) - \frac{g'\sin\theta}{g\sqrt{3}}(T_{9f}C_W - 2q_f S_W) \\
g(f)_{2V} &= -\sin\theta(T_{3f} - 2S_W^2 q_f) - \frac{g'\cos\theta}{g\sqrt{3}}(T_{9f}C_W - 2q_f S_W) \\
g(f)_{1A} &= \cos\theta T_{3f} - \frac{g'\sin\theta}{g\sqrt{3}}T_{9f}C_W \\
g(f)_{2A} &= -\sin\theta T_{3f} - \frac{g'\cos\theta}{g\sqrt{3}}T_{9f}C_W,
\end{aligned}
\tag{10}
$$

to be compared with the SM values $g(f)_{1V}^{SM} = T_{3f} - 2S_W q_f$ and $g(f)_{1A}^{SM} = T_{3f}$. The values of $g_{iV},\ g_{iA};\ \ i = 1, 2$ are listed in Tables II and III. As we can see, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM. Because of this, we can test the new phenomenology beyond the SM.

TABLE II. The $Z_1^\mu \longrightarrow \bar{f}f$ couplings.

f	g_{1V}	g_{1A}
u	$(\frac{1}{2}-\frac{4S_W^2}{3})[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$	$\frac{\cos\theta}{2}-\sin\theta/[2(4C_w^2-1)^{1/2}]$
d	$\cos\theta(-\frac{1}{2}+\frac{2S_W^2}{3})-\frac{\sin\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-\frac{S_W^2}{3})$	$-\frac{1}{2}\{\cos\theta+\sin\theta C_{2W}/[2(4C_W^2-1)^{1/2}]\}$
D	$\frac{2S_W^2\cos\theta}{3}+\sin\theta(1-\frac{5}{3}S_W^2)/(4C_W^2-1)^{1/2}$	$C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
e^-	$\cos\theta(-\frac{1}{2}+2S_W^2)+\frac{3\sin\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-S_W^2)$	$-\frac{\cos\theta}{2}+\frac{\sin\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-C_W^2)$
E^-	$\cos\theta(-1+2S_W^2)-\frac{S_W^2\sin\theta}{(4C_W^2-1)^{1/2}}$	$C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
$\nu_e,\ N_2^0$	$\frac{1}{2}[\cos\theta+\sin\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$	$\frac{1}{2}(\cos\theta+\sin\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}$
$N_1^0,\ N_3^0$	$-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$	$-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
N_4^0	$-\frac{1}{2}[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$	$-\frac{1}{2}[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$

TABLE III. The $Z_2^\mu \longrightarrow \bar{f}f$ couplings.

f	g_{2V}	g_{2A}
u	$(\frac{1}{2}-\frac{4S_W^2}{3})[-\sin\theta-\cos\theta/(4C_W^2-1)^{1/2}]$	$\frac{-\sin\theta}{2}-\cos\theta/[2(4C_w^2-1)^{1/2}]$
d	$-\sin\theta(-\frac{1}{2}+\frac{2S_W^2}{3})-\frac{\cos\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-\frac{S_W^2}{3})$	$-\frac{1}{2}\{-\sin\theta+\cos\theta C_{2W}/[2(4C_W^2-1)^{1/2}]\}$
D	$\frac{-2S_W^2\sin\theta}{3}+\cos\theta(1-\frac{5}{3}S_W^2)/(4C_W^2-1)^{1/2}$	$C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
e^-	$-\sin\theta(-\frac{1}{2}+2S_W^2)+\frac{3\cos\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-S_W^2)$	$\frac{\sin\theta}{2}+\frac{\cos\theta}{(4C_W^2-1)^{1/2}}(\frac{1}{2}-C_W^2)$
E^-	$-\sin\theta(-1+2S_W^2)-\frac{S_W^2\cos\theta}{(4C_W^2-1)^{1/2}}$	$C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
$\nu_e,\ N_2^0$	$\frac{1}{2}[-\sin\theta+\cos\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$	$\frac{1}{2}(-\sin\theta+\cos\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}$
$N_1^0,\ N_3^0$	$-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$	$-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
N_4^0	$\frac{1}{2}[\sin\theta+\cos\theta/(4C_W^2-1)^{1/2}]$	$\frac{1}{2}[\sin\theta+\cos\theta/(4C_W^2-1)^{1/2}]$

5 Masses for fermions

Again this subject is model dependent. Just for the sake of completeness let us write the Yukawa lagrangian that the three higgs scalars in Section **3** produce for

the fermion fields in model **A**[8]:

$$\begin{aligned}
\mathcal{L}_Y &= \mathcal{L}_Y^Q + \mathcal{L}_Y^l \\
\mathcal{L}_Y^Q &= \chi_L^T C(h_u \phi_3 u_L^c + h_D \phi_1 D_L^c + h_d \phi_2 d_L^c + h_{dD}\phi_2 D_L^c + h_{Dd}\phi_1 d_L^c) + h.c. \qquad (11) \\
\mathcal{L}_Y^l &= \epsilon_{abc}[\psi_L^a C(h_1 \psi_{1L}^b \phi_3^c + h_2 \psi_{2L}^b \phi_1^c + h_3 \psi_{2L}^b \phi_2^c) + \psi_{1L}^a C(h_4 \psi_{2L}^b \phi_1^c + h_5 \psi_{2L}^b \phi_2^c)] \\
&\quad + H.c., \qquad (12)
\end{aligned}$$

where $h_\eta,\ \eta = u, d, D, dD, Dd, 1, 2, 3, 4, 5$ are Yukawa couplings of order one, a, b, c are $SU(3)_L$ tensor indices and C is the charge conjugation operator.

Using the VEV as in section 3 and assuming that we are referring to the third family, we see that $m_t = h_u v'/\sqrt{2}$, $m_D \sim h_D V$ but it mixes with the b quark producing a kind of see-saw mechanism[13] that implies $m_b << m_t$. Also for leptons we have $m_E \sim h_4 V$ but again it mixes with the τ lepton producing also a kind of see saw mechanism which implies that $m_\tau \sim m_b << m_t$. The neutral sector is more complicated; the analysis of the 5×5 mass matrix gives: first two eigenvalues $\pm h_1 v'/\sqrt{2}$ which correspond to a Dirac neutrino with a mass of the order of the electroweak mass scale; other two are $\pm V + \eta$, where η is a small see-saw quotient, which correspond to a very massive pseudo-Dirac neutrino, and finally a tiny mass Majorana neutrino.

So the higgs fields and VEV used break the symmetry in the appropriate way, and produce a realistic pattern of masses for the fermion fields (at least for one of the families).

6 Conclusions

In this paper we have studied the theory of $SU(3)c \otimes SU(3)_L \otimes U(1)_X$ in detail. By restricting the fermion field representations to particles without exotic electric charges we end up with six different models, two one family models and four models for three families. The two one family models are sketched in the papers by K.T. Mahanthappa and P.K. Mohapatra in Ref.[4], but enough attention was not paid to the anomaly cancellation constraints in their analysis. The four three

family models are relatively new in the literature, with two of them (models **E** and **F**) introduced here for the first time, as far as we know.

If we allow for particles with exotic electric charges in our analysis, we end up with an infinite number of models, where the model in Refs.[5] is just one of them (probably the most elegant one!).

The low energy predictions of the six models discussed here are not the same. All of them have in common a new neutral current which mixes with the SM neutral current which is also included as part of each model. When the mixing angle between the two neutral currents is zero ($\sin\theta = 0$), exact agreement with the SM predictions is achieved, but the use of experimental results from LEP, SLAC and atomic parity violation bound the mixing angle to values which are model dependent. For partial analysis see for example Refs.[6, 8] and [9].

Detailed analysis in each model of flavor changing neutral currents, GIM mechanism, mass scales of the new gauge bosons, mass spectrum for the neutral spin 1/2 particles etc., are model dependent and they will be presented elsewhere.

Finally let us mention that the most remarkable result of our analysis is the existence of models **E** and **F**, where the three families are treated different. In these models it should be simple to implement the horizontal survival hypothesis[14], that is, to provide masses at tree level only for the particles in the third family, as done for example in the previous section, with the known particles in the first and second families getting masses as radiative corrections.

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